2 - 15 Systems of ODEs
Using the Laplace transform and showing the details of your work, solve the IVP:

3. $y_1' = -y_1 + 4 y_2$, $y_2' = 3 y_1 - 2 y_2$, $y_1[0] = 3$, $y_2[0] = 4$

```
ClearAll["Global`*"]

e1 = DSolve[{y1'[t] == -y1[t] + 4 y2[t],
   y2'[t] == 3 y1[t] - 2 y2[t], y1[0] == 3, y2[0] == 4}, {y1, y2}, t]
```

$$\{\{y1 \to \text{Function}[\{t\}, e^{-5t} (-1 + 4 e^{7t})], y2 \to \text{Function}[\{t\}, e^{-5t} (1 + 3 e^{7t})]\}\}$$

Above: The answer matches the text's. It seems that **DSolve** is able to do this without any reference to Laplace.

5. $y_1' = y_2 + 1 - u(t-1)$, $y_2' = -y_1 + 1 - u(t-1)$, $y_1[0] = 0$, $y_2[0] = 0$

```
ClearAll["Global`*"]

e1 = DSolve[{y1'[t] == y2[t] + 1 - UnitStep[t - 1],
   y2'[t] == -y1[t] + 1 - UnitStep[t - 1], y1[0] == 0, y2[0] == 0}, {y1, y2}, t]

{{y1 → Function[{t}, -Cos[t] + Cos[t]^2 + Sin[t] + Sin[t]^2 +
     Cos[1] Cos[t] UnitStep[-1 + t] - Cos[t]^2 UnitStep[-1 + t] +
     Cos[t] Sin[1] UnitStep[-1 + t] - Cos[1] Sin[t] UnitStep[-1 + t] +
     Sin[1] Sin[t] UnitStep[-1 + t] - Sin[t]^2 UnitStep[-1 + t]],
  y2 → Function[{t}, Cos[t] - Cos[t]^2 + Sin[t] - Sin[t]^2 -
     Cos[1] Cos[t] UnitStep[-1 + t] + Cos[t]^2 UnitStep[-1 + t] +
     Cos[t] Sin[1] UnitStep[-1 + t] - Cos[1] Sin[t] UnitStep[-1 + t] -
     Sin[1] Sin[t] UnitStep[-1 + t] + Sin[t]^2 UnitStep[-1 + t]]}}

e2 = e1[[1, 1, 2, 2]]

-Cos[t] + Cos[t]^2 + Sin[t] + Sin[t]^2 +
 Cos[1] Cos[t] UnitStep[-1 + t] - Cos[t]^2 UnitStep[-1 + t] +
 Cos[t] Sin[1] UnitStep[-1 + t] - Cos[1] Sin[t] UnitStep[-1 + t] +
 Sin[1] Sin[t] UnitStep[-1 + t] - Sin[t]^2 UnitStep[-1 + t]

e3 = Collect[e2, UnitStep[-1 + t]]

-Cos[t] + Cos[t]^2 + Sin[t] + Sin[t]^2 +
 (Cos[1] Cos[t] - Cos[t]^2 + Cos[t] Sin[1] -
   Cos[1] Sin[t] + Sin[1] Sin[t] - Sin[t]^2) UnitStep[-1 + t]
```

Some hand substitutions to shape the expression to look more like the text answer.

```
e4 = e3 /. Cos[t]^2 + Sin[t] + Sin[t]^2 → 1 + Sin[t]
```

```
1 - Cos[t] + Sin[t] + (Cos[1] Cos[t] - Cos[t]^2 + Cos[t] Sin[1] -
     Cos[1] Sin[t] + Sin[1] Sin[t] - Sin[t]^2) UnitStep[-1 + t]
```

And some more.

```
e5 = e4 /. -Cos[t]^2 + Cos[t] Sin[1] - Cos[1] Sin[t] + Sin[1] Sin[t] - Sin[t]^2 ->
    -1 + Cos[t] Sin[1] - Cos[1] Sin[t] + Sin[1] Sin[t]
```

```
1 - Cos[t] + Sin[t] +
 (-1 + Cos[1] Cos[t] + Cos[t] Sin[1] - Cos[1] Sin[t] + Sin[1] Sin[t])
  UnitStep[-1 + t]
```

Working in a couple of trig identities.

```
FullSimplify[Sin[1 - t] == Sin[1] Cos[t] - Cos[1] Sin[t]]
```

```
True
```

```
e6 = e5 /. Cos[t] Sin[1] - Cos[1] Sin[t] → Sin[1 - t]
```

```
1 - Cos[t] + Sin[t] +
 (-1 + Cos[1] Cos[t] + Sin[1 - t] + Sin[1] Sin[t]) UnitStep[-1 + t]
```

```
FullSimplify[Cos[t - 1] == Cos[t] Cos[1] + Sin[t] Sin[1]]
```

```
True
```

```
e7 =
 e6 /. (Cos[1] Cos[t] + Sin[1 - t] + Sin[1] Sin[t]) → (Cos[t - 1] + Sin[1 - t])
```

```
1 - Cos[t] + Sin[t] + (-1 + Cos[1 - t] + Sin[1 - t]) UnitStep[-1 + t]
```

```
PossibleZeroQ[
 (1 - Cos[t] + Sin[t] + (-1 + Cos[1 - t] + Sin[1 - t]) UnitStep[-1 + t]) -
  (-Cos[t] + Sin[t] + 1 + UnitStep[-1 + t] (-1 + Cos[t - 1] - Sin[t - 1]))]
```

```
True
```

Above: The answer matches the text answer (for y_1) in content, as shown by the PZQ above.

And a couple more trig identities.

```
Sin[1 - x] == -Sin[x - 1]
```

```
True
```

```
Cos[1 - x] == Cos[x - 1]
```

```
True
```

```
e8 = e1[[1, 2, 2, 2]]
Cos[t] - Cos[t]^2 + Sin[t] - Sin[t]^2 -
 Cos[1] Cos[t] UnitStep[-1 + t] + Cos[t]^2 UnitStep[-1 + t] +
 Cos[t] Sin[1] UnitStep[-1 + t] - Cos[1] Sin[t] UnitStep[-1 + t] -
 Sin[1] Sin[t] UnitStep[-1 + t] + Sin[t]^2 UnitStep[-1 + t]

e9 = Collect[e8, UnitStep[-1 + t]]
Cos[t] - Cos[t]^2 + Sin[t] - Sin[t]^2 +
 (-Cos[1] Cos[t] + Cos[t]^2 + Cos[t] Sin[1] -
    Cos[1] Sin[t] - Sin[1] Sin[t] + Sin[t]^2) UnitStep[-1 + t]

e10 =
 e9 /. (Cos[t]^2 + Cos[t] Sin[1] - Cos[1] Sin[t] - Sin[1] Sin[t] + Sin[t]^2) →
   (1 + Cos[t] Sin[1] - Cos[1] Sin[t] - Sin[1] Sin[t])
Cos[t] - Cos[t]^2 + Sin[t] - Sin[t]^2 +
 (1 - Cos[1] Cos[t] + Cos[t] Sin[1] - Cos[1] Sin[t] - Sin[1] Sin[t])
  UnitStep[-1 + t]

e11 = e10 /. -Cos[t]^2 + Sin[t] - Sin[t]^2 -> -1 + Sin[t]
-1 + Cos[t] + Sin[t] +
 (1 - Cos[1] Cos[t] + Cos[t] Sin[1] - Cos[1] Sin[t] - Sin[1] Sin[t])
  UnitStep[-1 + t]

e12 = e11 /. -Cos[1] Cos[t] + Cos[t] Sin[1] - Cos[1] Sin[t] - Sin[1] Sin[t] ->
   -Cos[1 - t] + Cos[t] Sin[1] - Cos[1] Sin[t]
-1 + Cos[t] + Sin[t] +
 (1 - Cos[1 - t] + Cos[t] Sin[1] - Cos[1] Sin[t]) UnitStep[-1 + t]

e13 = e12 /. Cos[t] Sin[1] - Cos[1] Sin[t] → Sin[1 - t]
-1 + Cos[t] + Sin[t] + (1 - Cos[1 - t] + Sin[1 - t]) UnitStep[-1 + t]

PossibleZeroQ[
 (-1 + Cos[t] + Sin[t] + (1 - Cos[1 - t] + Sin[1 - t]) UnitStep[-1 + t]) -
  (Cos[t] + Sin[t] - 1 + UnitStep[-1 + t] (1 - Cos[t - 1] - Sin[t - 1]))]
True
```

Above: The answer in green matches the text answer (for y_2) in content, as shown by the PZQ above.

7. $y_1' = 2y_1 - 4y_2 + u(t-1)e^t$, $y_2' = y_1 - 3y_2 + u(t-1)e^t$, $y_1[0] = 3$, $y_2[0] = 0$

```
ClearAll["Global`*"]
```

```
e1 = DSolve[{y1'[t] == 2 y1[t] - 4 y2[t] + UnitStep[t - 1] ⅇ^t, y2'[t] ==
    y1[t] - 3 y2[t] + UnitStep[t - 1] ⅇ^t, y1[0] == 3, y2[0] == 0}, {y1, y2}, t]
```

$$\{\{y1 \to \text{Function}[\{t\}, \frac{1}{3} e^{-2t} (-3 + 12 e^{3t} - e^3 \text{UnitStep}[-1+t] + e^{3t} \text{UnitStep}[-1+t])],$$
$$y2 \to \text{Function}[\{t\}, \frac{1}{3} e^{-2t} (-3 + 3 e^{3t} - e^3 \text{UnitStep}[-1+t] + e^{3t} \text{UnitStep}[-1+t])]\}\}$$

```
e2 = e1[[1, 1, 2, 2]]
```

$$\frac{1}{3} e^{-2t} (-3 + 12 e^{3t} - e^3 \text{UnitStep}[-1+t] + e^{3t} \text{UnitStep}[-1+t])$$

```
e3 = Collect[e2, UnitStep[-1 + t]]
```

$$\frac{1}{3} e^{-2t} (-3 + 12 e^{3t}) + \frac{1}{3} e^{-2t} (-e^3 + e^{3t}) \text{UnitStep}[-1+t]$$

$$\text{PossibleZeroQ}\left[\left(\frac{1}{3} e^{-2t} (-3 + 12 e^{3t}) + \frac{1}{3} e^{-2t} (-e^3 + e^{3t}) \text{UnitStep}[-1+t]\right) - \left(-e^{-2t} + 4 e^t + \frac{1}{3} \text{UnitStep}[-1+t] (-e^{3-2t} + e^t)\right)\right]$$

True

Above: This answer matches the text in content (y_1), as shown by the PZQ above.

```
e4 = e1[[1, 2, 2, 2]]
```

$$\frac{1}{3} e^{-2t} (-3 + 3 e^{3t} - e^3 \text{UnitStep}[-1+t] + e^{3t} \text{UnitStep}[-1+t])$$

```
e5 = Collect[e4, UnitStep[-1 + t]]
```

$$\frac{1}{3} e^{-2t} (-3 + 3 e^{3t}) + \frac{1}{3} e^{-2t} (-e^3 + e^{3t}) \text{UnitStep}[-1+t]$$

$$\text{PossibleZeroQ}\left[\left(\frac{1}{3} e^{-2t} (-3 + 3 e^{3t}) + \frac{1}{3} e^{-2t} (-e^3 + e^{3t}) \text{UnitStep}[-1+t]\right) - \left(-e^{-2t} + e^t + \frac{1}{3} \text{UnitStep}[-1+t] (-e^{3-2t} + e^t)\right)\right]$$

True

Above: This answer matches the text in content (y_2), as shown by the PZQ above.

9. $y_1' = 4 y_1 + y_2,\ y_2' = -y_1 + 2 y_2,\ y_1[0] = 3,\ y_2[0] = 1$

```
ClearAll["Global`*"]
```

```
e1 = DSolve[{y1'[t] == 4 y1[t] + y2[t],
   y2'[t] == -y1[t] + 2 y2[t], y1[0] == 3, y2[0] == 1}, {y1, y2}, t]
```

```
{{y1 → Function[{t}, e^(3 t) (3 + 4 t)], y2 → Function[{t}, -e^(3 t) (-1 + 4 t)]}}
```

Above: The answer matches the text answer.

11. $y_1'' = y_1 + 3 y_2$, $y_2'' = 4 y_1 - 4 e^t$, $y_1[0] = 2$,
$y_1[0] = 2$, $y_1'[0] = 3$, $y_2[0] = 1$, $y_2'[0] = 2$

```
ClearAll["Global`*"]
```

```
e1 = DSolve[{y1''[t] == y1[t] + 3 y2[t], y2''[t] == 4 y1[t] - 4 e^t,
   y1[0] == 2, y1'[0] == 3, y2[0] == 1, y2'[0] == 2}, {y1, y2}, t]
```

$$\{\{y1 \to \text{Function}[\{t\}, \frac{1}{7} e^t (4 + 7 e^t + 3 \text{Cos}[\sqrt{3}\, t]^2 + 3 \text{Sin}[\sqrt{3}\, t]^2)],$$
$$y2 \to \text{Function}[\{t\}, \frac{1}{7} e^t (4 + 7 e^t - 4 \text{Cos}[\sqrt{3}\, t]^2 - 4 \text{Sin}[\sqrt{3}\, t]^2)]\}\}$$

```
e2 = e1[[1, 1, 2, 2]]
```

$$\frac{1}{7} e^t (4 + 7 e^t + 3 \text{Cos}[\sqrt{3}\, t]^2 + 3 \text{Sin}[\sqrt{3}\, t]^2)$$

```
e3 = FullSimplify[e2]
```

$$e^t (1 + e^t)$$

Above: The answer matches the text answer (y_1).

```
e4 = e1[[1, 2, 2, 2]]
```

$$\frac{1}{7} e^t (4 + 7 e^t - 4 \text{Cos}[\sqrt{3}\, t]^2 - 4 \text{Sin}[\sqrt{3}\, t]^2)$$

```
e5 = FullSimplify[e4]
```

$$e^{2 t}$$

Above: The answer matches the text answer (y_2).

13. $y_1'' + y_2 = -101 \text{Sin}[10 t]$, $y_2'' + y_1 = 101 \text{Sin}[10 t]$,
$y_1[0] = 0$, $y_1'[0] = 6$, $y_2[0] = 8$, $y_2'[0] = -6$

```
ClearAll["Global`*"]
```

```
e1 =
 DSolve[{y1''[t] + y2[t] == -101 Sin[10 t], y2''[t] + y1[t] == 101 Sin[10 t],
   y1[0] == 0, y1'[0] == 6, y2[0] == 8, y2'[0] == -6}, {y1, y2}, t]
```

```
{{y1 → Function[{t}, -4 e^t + 4 Cos[t] + Sin[10 t]],
  y2 → Function[{t}, 4 e^t + 4 Cos[t] - Sin[10 t]]}}
```

Above: The answers match the text answers (y_1 & y_2).

15. $y_1' + y_2' = 2\,\text{Sinh}[t]$, $y_2' + y_3' = e^t$,
$y_3' + y_1' = 2\,e^t + e^{-t}$, $y_1[0] = 1$, $y_2[0] = 1$, $y_3[0] = 0$

```
ClearAll["Global`*"]
```

```
e1 = DSolve[{y1'[t] + y2'[t] == 2 Sinh[t], y2'[t] + y3'[t] == e^t,
   y3'[t] + y1'[t] == 2 e^t + e^-t, y1[0] == 1, y3[0] == 0}, {y1, y2, y3}, t]
```

```
{{y1 → Function[{t}, e^t], y2 → Function[{t}, e^-t (1 + e^t C[2])],
  y3 → Function[{t}, e^-t (-1 + e^(2 t))]}}
```

Above: The answers match those of the text for y_1, y_2, and y_3 (with choice of constant = 0 for C[2]).